Wave

Wave is a disturbance or energy propogates in the medium due to repeated periodic motion of the particle about their mean position. i.e. the disturbance is handled from particle to particle & this process is called wave motion.

On the basis of requirement of material medium, there are three types of wave:-

1) Electromagnetic wave: The wave which doesnt require any material medium for its propagation is called electromagnetic wave.
Eg: light wave, heat wave, x-rays & γ-rays

2) Mechanical wave: The wave which requires a material medium for its propogation is called mechanical wave or elastic wave.
Eg: Sound wave, ripples in pond, wave in a stretched string

3) Particle wave: The wave associated with the microscopic particle such as electron, proton, neutron etc, which they are in motion, the wave produced is called matter wave or debroglie wave

Mechanical Wave

On the basis of vibration of the particle of the ~~part~~ medium. The waves are of two types:

1) Transverse waves: If the particles of the medium vibrate ⊥r to the direction of propagation of wave, then the wave is called transverse wave.
Eg: Wave in a stretched string, Ripples in pond

These waves propagates in the medium in the form of alternative crest & trough.

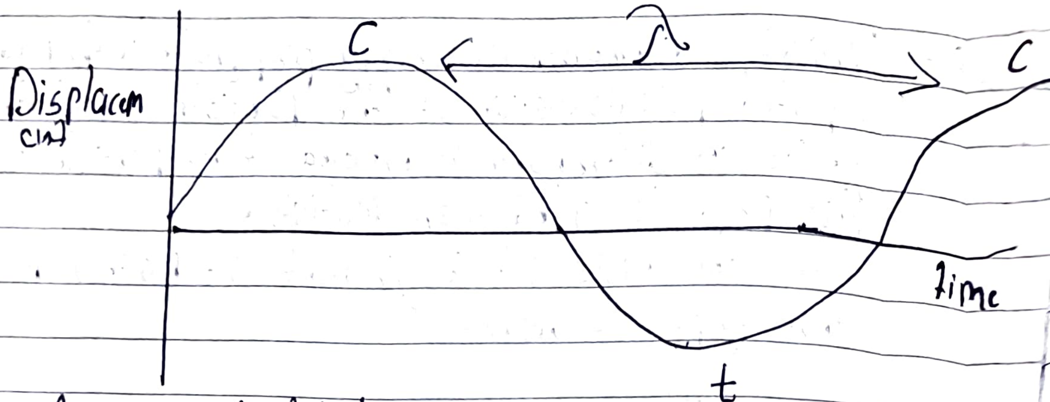


Fig: Wave in stretched spring

- Crest: The maximum upward displacement of the particle from the normal line is called crest.
  Trough: The maximum downward displacement of the particle from the normal line is called ~~crest~~ trough.
- The distance between two consecutive crest or two consecutive trough is consecutive trough is called wave length (λ).
- It can travel through the solid & surface of liquid.
- It can be polarized.

= Longitudinal: If the particles of the medium vibrate along the direction of propagation of wave, then the wave is called longitudinal wave.

Eg: Sound wave, Spring wave

These waves propagates in the medium in the form of alt compression & rarefaction

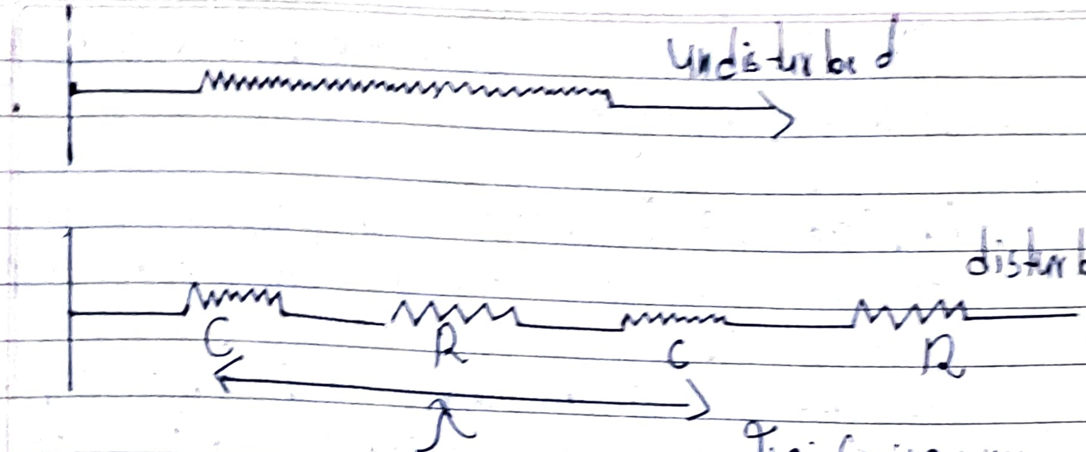


Fig: Spring wave

Compression: Those point in the medium where the pressure & density is maximum is called compression.

Rarefaction: Those point in the medium where pressure & density is minimum.

The distance between two consecutive compression or rarefaction is called wave length.

It can travel through solid, liquid & gas.
It can't be polarized.

(Q) Diff btwn Sun & light wave: ?

For the propagation in the medium it must posses the property of elasticity (to store the P.E) & property of inertia. (To store K.E as it is constant).

★ Basic Terminology

(1) wave length (λ): Distance travelled by the wave in 1 complete cycle/oscillation is called wave length.

OR: The distance between the two particle vibrating in the same phase is wave length

(2) Amplitude (a): The maximum displacement of particle about their mean position in the medium is called Amplitude.

(3) Frequency (f): The number of oscillation completed by the wave in the medium in one second is called frequency.

i.e $f = \frac{1}{T}$

(4) Time period (T): The time during which a particle in the medium completes one oscillation is called time period.

(5) Wave ~~length~~ velocity (v): The linear distance travelled by the wave in the medium per unit time along the propagation of wave is called wave velocity.

i.e

Wave velocity (v) = $\frac{\text{Distance travelled along the propagation of wave (along x-axis)}}{\text{Time taken}}$

$v = \frac{\text{wave length}}{\text{Time period}}$ $\left[\lambda \propto \frac{1}{T}\right]$

$v = \frac{\lambda}{T}$

$v = \lambda f$ $\left(f = \frac{1}{T}\right)$

( ) Partical velocity (V_0): The velocity of the vibrating particle when the wave is passing through it is called partical velocity (Ve)

$v = \omega\sqrt{a^2 - x^2}$

If $x = 0$ then

$v_{max} = \omega a$

(7) **Phase (ϕ):** The phase of the wave represents the current position of the wave with reference position. It also describes the state of motion of particle in a given point & time.

Relation between Phase & Path diff^n

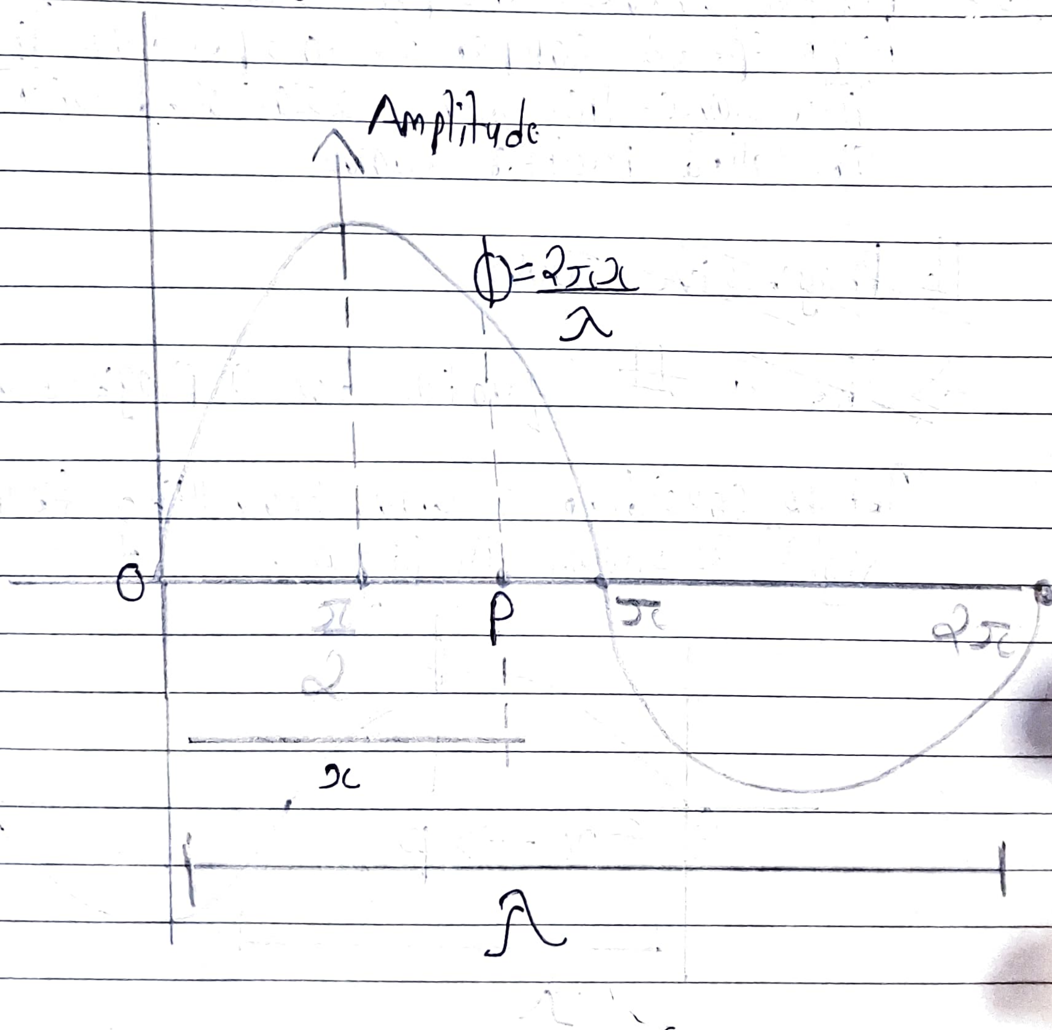
For path diff λ, the phase diff $(\phi) = 2\pi$

For path diff 1, the phase diff $(\phi) = \frac{2\pi}{\lambda}$

For path diff x, the phase diff $(\phi) = \frac{2\pi x}{\lambda}$

$$\phi = \frac{2\pi x}{\lambda}$$

$$\Delta\phi = \frac{2\pi \Delta x}{\lambda}$$



Progressive Wave: A progressive wave is one in which disturbance travels continuously along the direction of propagation of wave.

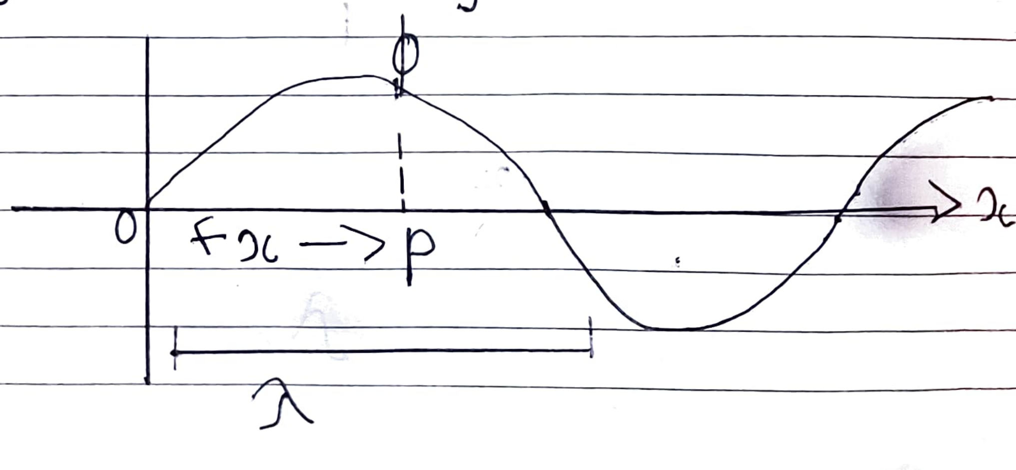
- The disturbance travel forward but they vibrate little lot then the particle before it.
- All the Particles vibrate with the same amplitude & frequency
- No particle is at rest but they vibrate with different phase.
- It may be transverse or longitudinal

Matter Waves: The particles like electron at very high speed, comparable to speed of light behave like waves. This waves associated with such particles are called matter waves.

~~Progressive~~ Wave

=> ~~The ω~~ # Equation for a progressive wave

Let us consider a wave travelling from left to right as shown in figure.



The displacement of a particle of the medium at the origin O is given by

$$y = a\sin\omega t \quad \text{—(i)}$$

where a is amplitude & ω is angular velocity

Now,

Let another particle at point P at a distance x from the origin O. If φ be the phase difference of the particle at P than its displacement is given by

$$y = a\sin(\omega t - \phi) \quad \text{—(ii)}$$

Now, Phase diff,

$$\phi = \frac{2\pi x}{\lambda}$$

Then eqn (ii) becomes

$$y = a\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

$$y = a\sin(\omega t - kx) \quad \text{—(iii)}$$

Where $k = \frac{2\pi}{\lambda}$ is called propagation constant or wave number

Now,

$$\omega = 2\pi f \quad \& \quad f = \frac{1}{T}$$

$$\therefore \omega = \frac{2\pi}{T}$$

then,

$$y = a\sin\left(\frac{2\pi}{T} \times t - \frac{2\pi x}{\lambda}\right)$$

$$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Since,

$$\omega = 2\pi f$$

Also, $v = f\lambda$

$$f = \frac{v}{\lambda}$$

Putting the value of f in $\omega = 2\pi f$

$$\omega = \frac{2\pi v}{\lambda}$$

$$\omega = \frac{2\pi v}{\lambda}$$

where v is speed of wave,

Putting the value of ω & k in eqn (iii)

$$y = a\sin\left(\frac{2\pi v}{\lambda}t - \frac{2\pi x}{\lambda}\right) \quad \text{(iv)}$$

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) \quad \text{(v)}$$

Eqn (iv) & (v) represents the plane progessive wave.

Equation for wave travelling in the opposite direction (i.e from right to left) is

$$y = a\sin\frac{2\pi}{\lambda}(vt + x)$$

Principle of superposition,

→ It states that, when two or more waves are passing through a medium at the same time, resultant displacement at any point is equal to the vector sum of their individual displacments at that point.

If $\vec{y_1}, \vec{y_2}, \vec{y_3} \dots y_n$ be the displacments at a point due to individual waves, then the resultant displacement is given by:

$$\vec{y} = \vec{y_1} + \vec{y_2} + \vec{y_3} + \dots + \vec{y_n}$$

anti → displacement max N = Node displacement 0

Stationary wave:- A wave formed by the superposition of two progressive waves of the same frequency & amplitude, travelling with same speed but in the opposite direction.

resultant displacement is 0
$y = y_1 - y_2$
$a - a$
$= 0$

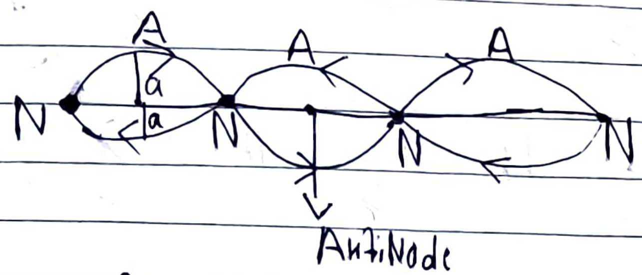


Fig: Stationary wave.

In a stationary wave, there are two types of points, they are:

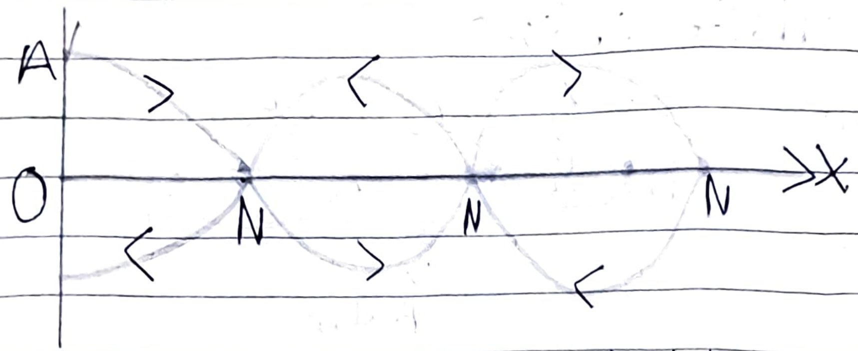
1. Nodes :- These are the points ~~that~~ at which particle displacement is 0. These points are denoted by N. (minimum)
2. AntiNodes :- These are the points at which particle displacement is maximum. These points are denoted by A.

MCQ Distance between two consecutive nodes is $\frac{\lambda}{2}$. & 3 consecutive λ.

$$N-N / A-A \Rightarrow \frac{\lambda}{2}$$

$$N-A \Rightarrow \frac{\lambda}{4}$$

Equation of a stationary wave:



Let us consider a wave along a stretched string. The displacement of a wave travelling from left to right is given by,

$$y_1 = a\sin(\omega t - kx) \quad \text{— (i)}$$

The displacement of reflected wave travelling in the opposite direction,

$$y_2 = a\sin(\omega t + kx) \quad \text{— (ii)}$$

a = amplitude

When these two waves superpose each other, a stationary wave is produced. The resultant displacement is given by principle of superposition as,

$$y = y_1 + y_2$$

$$y = a\sin(\omega t - kx) + a\sin(\omega t + kx)$$

$$a\left[\sin(\omega t - kx) + \sin(\omega t + kx)\right]$$

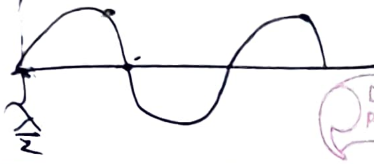
$$a\left[2\sin\frac{\omega t - kx + \omega t + kx}{2} \cdot \cos\frac{\omega t - kx - \omega t - kx}{2}\right]$$

$$2a\left[\sin \omega t \cdot \cos kx\right]$$

$$y = 2a\cos kx \cdot \sin \omega t \quad \text{— (iii)}$$

Eq^n - (3) gives the equation of a stationary wave

$\frac{\pi}{2} + x$



Also, we can write, $y = A \sin \omega t$ — (iv)

where, $A = 2a \cos kx$ is the amplitude of resultant stationary wave.

For nodes,

for $y = 0$

i.e $A = 0$

$$2a \cos kx = 0$$
$$\cos kx = 0$$
$$\cos kx = \cos\left(n + \frac{1}{2}\right)\pi$$
$$kx = \left(n + \frac{1}{2}\right)\pi$$
$$\frac{2\pi}{\lambda} x = \left(n + \frac{1}{2}\right)\pi$$
$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$$

where $n = 0, 1, 2 \ldots$

If $n = 0$

$$x_0 = \frac{\lambda}{4}$$

If $n = 1$

$$x_1 = \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{3\lambda}{4}$$

Distance between two consecutive nodes,

$$x_1 - x_0$$
$$= \frac{3\lambda}{4} - \frac{\lambda}{4}$$
$$= \boxed{\frac{\lambda}{2}}$$

For antinodes,

y = Maximum

For max value of y,

$y = A \sin \omega t$

~~$\sin \omega t$ A should be 1~~

~~$y = A$~~

~~$y = 2a \cos kx$~~

~~$\cos kx$~~

$\cos kx = \pm 1$

$\cos kx = \cos n\pi$

where, $n = 0, 1, 2, 3, \ldots$

$kx = n\pi$

$\frac{2\pi}{\lambda} x = n\pi$

$x = \frac{n\lambda}{2}$

If $n = 0$

$x_0 = 0$

If $n = 1$

$x_1 = \frac{\lambda}{2}$

Distance between two consecutive antinodes $= x_1 - x_0$

$\frac{\lambda}{2} - 0 = \frac{\lambda}{2}$

Q) If the velocity of sound in air is 340 m/s. Calculate

a) Wave length when frequency is 256 hz

b) The frequency when wave length is 0.85m

$-\hat{c} =$ CH_2 — CH_2 CH_2 CH_2 CH_2 C_5H_{13} CH_2

$CH_3 - CH_2 - CH_2 - CH_2 - CH_3$

(a) $V = \lambda f$

$340 = \lambda \times 256$

$\lambda = 1.32m$

(b) $340 = 0.85f$

$f = 400hz$

(a) For small piece of cork in a ripple tank oscillates up down as repulse pass it. If the repulse travel at 0.20m/s at wave length of 15mm & an amplitude of 5.0mm, what is the maximum velocity of the cork?

=> $V = 0.20m/s$

$\lambda = 15mm = 15 \times 10^{-3} m$

$A = 5 \times 10^{-3} m$

For

Date
Page

(Q) Distance between two consecutives waves at the same phase?

-> λ

(Q) Which wave can propogate in liquids.

-> Only longitudinal wave can propogate in liquid. the liquid lacks the necessary rigidity to support the transverse wave.

Note: Sound waves travelling through rocks are the combination of longitudinal wave & transverse wave while sound travels as longitudinal wave in air & liquid. Rocks (solid) have the ability to resist the change in the shape.

\# $y = \cos\left(\frac{2\pi}{60x}\right)\sin at$

x, y in cm & t in sec

At node, xs?

(a) $\frac{100}{2\pi}$ cm, (b) 20cm (c) 15cm (d) 12.5cm

-> $y = \cos\left(\frac{2\pi}{60x}\right)$ $\boxed{\sin at}$ — is constant

$0 = \cos\left(\frac{2\pi}{60x}\right)$

$\cos 90 = \cos\frac{2\pi}{60x}$

$\frac{2\pi}{3060}$ ~~$\frac{2\pi}{3060}$~~ $x = \frac{\pi}{2}$

$$x = \frac{30}{2} = 15\,cm$$

(Q)

$x_1 = a\sin(\omega t + \phi_1)$

$x_2 = a\sin(\omega t + \phi_2)$

Phase $\delta = ?$

$$x = x_1 + x_2$$

$$a[\sin(\omega t + \phi) + \sin(\omega t + \phi)]$$

$$\frac{2a}{a}\sin\left(\omega t + \frac{\phi_1 + \phi_2}{2}\right)\cos\frac{\phi_1 - \phi_2}{2}$$

$$2a\cos\frac{\phi_1 - \phi_2}{2} = a$$

$$\cos\frac{\phi_1 - \phi_2}{2} = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\frac{\phi_1 - \phi_2}{2} = \frac{\pi}{3}$$

$$\phi_1 - \phi_2 = \frac{2\pi}{3}$$

$$R = \sqrt{a^2 + a^2 + 2a \cdot a \cos\theta}$$

$$a^2 = 2a^2(1 + \cos\theta)$$

$$1 + \cos\theta = \frac{1}{2}$$

$$\cos\theta = \frac{-1}{2} = \cos 120$$

$$\theta = 120^0$$