Surface Tension

→ The property of liquid at rest by virtue of which its surface tries to occupy minimum possible surface area & behaves like stretched membrane is called surface tension.

→ If F is the force acting on the imaginary line of length l then surface tension is given by

$$T = \frac{F\ (\text{Force})}{l\ (\text{length})}$$

Its SI unit is Nm^{-1}.
Its dimensional formula is $[ML^0T^{-2}]$.

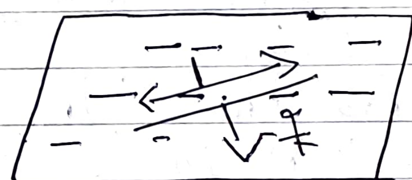


Fig: Surface Tension

Surface Energy :- The potential energy per unit area of the surface film is called surface energy. It is also defined as the amount of workdone in Increasing the area of surface film throug h unity.

It is denoted by σ & given by

$$\sigma = \frac{\text{Workdone on increasing the surface area (W)}}{\text{Threre Increasing surface area } (\Delta A)}$$

$$\sigma = \frac{W}{\Delta A}$$

Relation between surface energy & surface Tension

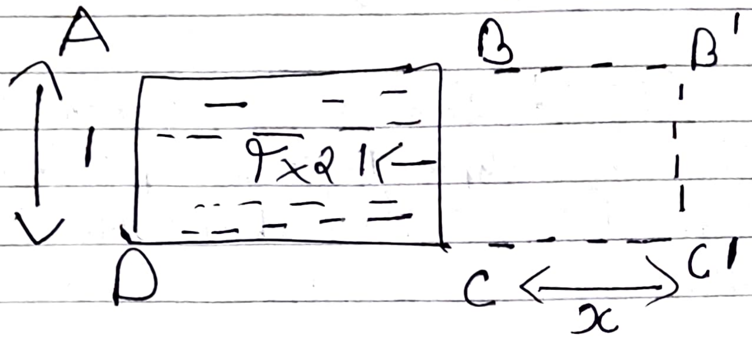


Fig: Surface Tension & Surface Energy.

→ Let us consider a rectangular frame of wire ABCD as shown in fig in which wire BC is movable. If we dip the frame in a soap solⁿ, a thin film is formed which pulls the wire BC towards left due to surface tension. Let T be the surface tension of film & l is the length of the wire BC then

Force (F) = $T \times 2l$

As the film has 2 surfaces in contact with wire so, length of wire it is taken as 2l.

Suppose the wire is moved through a distance x from BC to B'C' against surface tension force so that surface area increases.

Workdone on increasing surface area (W) = Force × displacement

$$W = T \times 2l \times x$$

&

Increase in surface area $= l \times x + l \times x$
$= 2lx$

Now,

surface energy $(\sigma) = \dfrac{W}{\Delta A}$

$$\sigma = \frac{T \times \cancel{2l} x}{\cancel{2lx}}$$

$$\boxed{\sigma = T}$$

Surface Tension is numerically equal to surface energy.

Excess Pressure on curve surface of liquid

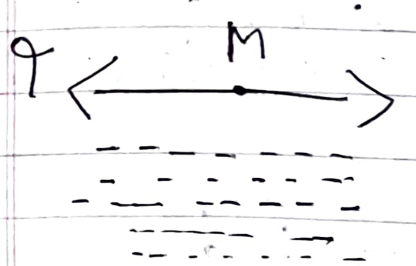


Fig (I)

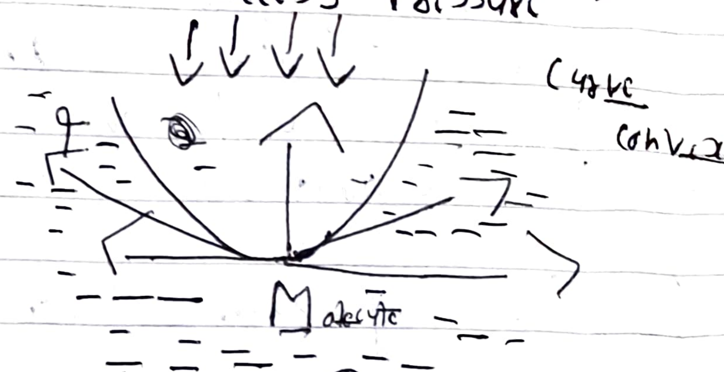
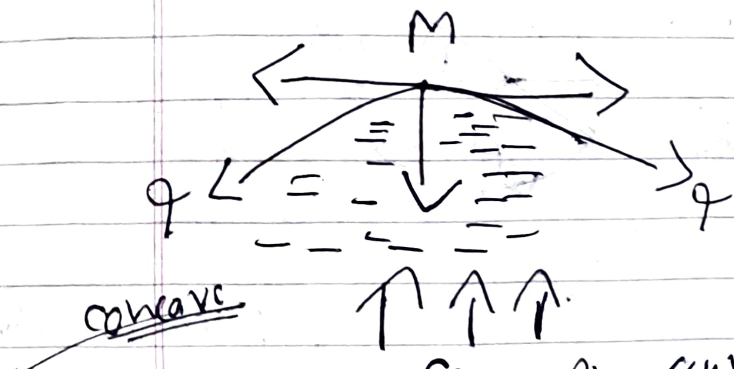


Fig (ii)

(H_2O) in glass.



Excess Pressure (Hg) in glass

Fig (iii)

→ If the liquid surface is plane the molecule M on the liquid surface is acted by force of surface tension tangentially to the liquid surface in opposite directions as shown in fig (i). Thus, the resultant force acting normal to the liquid surface is zero. If the liquid surface is curved, the molecule M on the liquid surface is acted upon forces due to surface tension along the tangent to the surface. Resolving these forces into horizontal & vertical components, horizontal component (T cos θ) cancel each other whereas vertical components (T sin θ) add up. Thus resultant force normal to the surface acts on the curved surface of liquid. For the equillibrium of the curved liquid surface, there must be an excess of pressure on its concave surface so that excess pressure force balances the resultant force due to surface tension.

Excess Pressure inside a liquid drop.

Let us consider a drop of liquid of radius ~~R~~ R as shown in figure if P is excess pressure inside the liquid drop & T is the surface tension

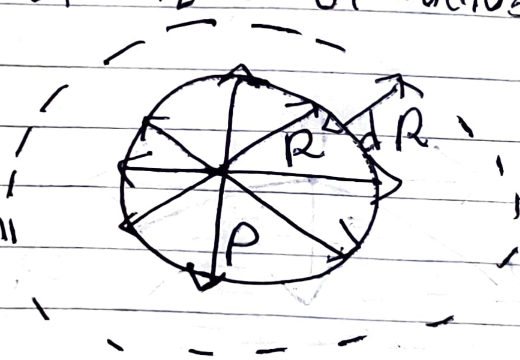


Fig: Excess Pressure inside a liquid drop.

Then,
work done by excess pressure = force × displacement

$$W = P \times A \times dR$$
$$= P\, 4\pi R^2 \times dR \quad \text{—(I)}$$

$$\boxed{P = \frac{T}{A}}$$

Now,
Increase in surface area
= final surface area - Initial surface area
$$4\pi(R + dR)^2 - 4\pi R^2$$
$$= \cancel{4\pi R^2} + 8\pi R\,dR + 4\pi\,dR^2 - \cancel{4\pi R^2}$$
$$\Delta A = 8\pi\,dR \cdot R$$

dR^2 is so small & it is neglected

Ans Here,

Workdone (W) = Increase in surface area $(\Delta A) \times T$.

$$W = 8\pi R\, dR \times T \quad \text{—— (II)}$$

From (i) & (ii)

$$P\, 4\pi R^2 \times dR = 8\pi R\, dR\, T$$

$$PR = 2T$$

air bubble → $\boxed{P = \frac{2T}{R}}$ (III)

[One surface soap/liquid
2 surface air contact $= \frac{4T}{R}$].

OR

$$P_{in} - P_{out} = \frac{2T}{R}$$

Angle of contact: (Capillary angle)

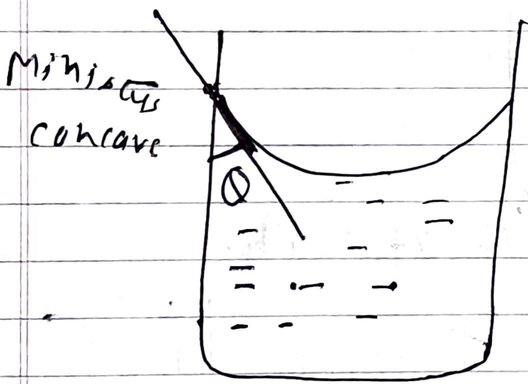


Fig: (I) Water in glass

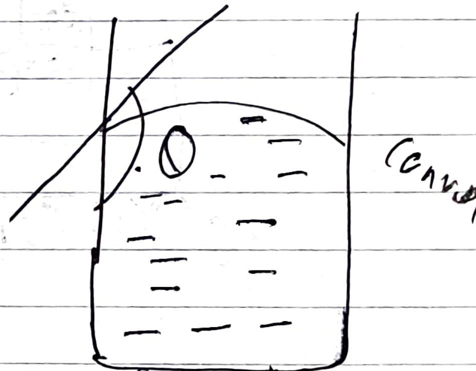


Fig:- Mercury in glass.

-> The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact or capillary angle. The angle of contact is acute in case of liquid which ~~width~~ wet wall of the container as shown in fig 1. & the angle of contact is obtuse in case of liquid which donot wet the wall of the container as shown in fig 2.

Capillary action or Capillarity.

The tube of very fine bore is called capillary tube. The rise or fall of liquid in a tube of very fine bore is called capillary action.

Fig (I): Capillary rise of water.

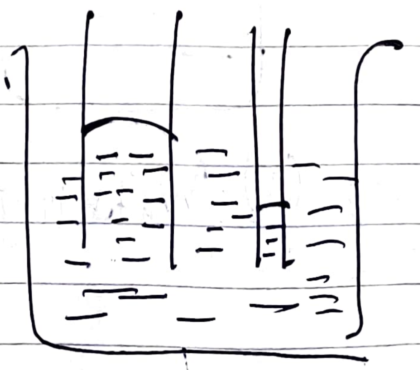
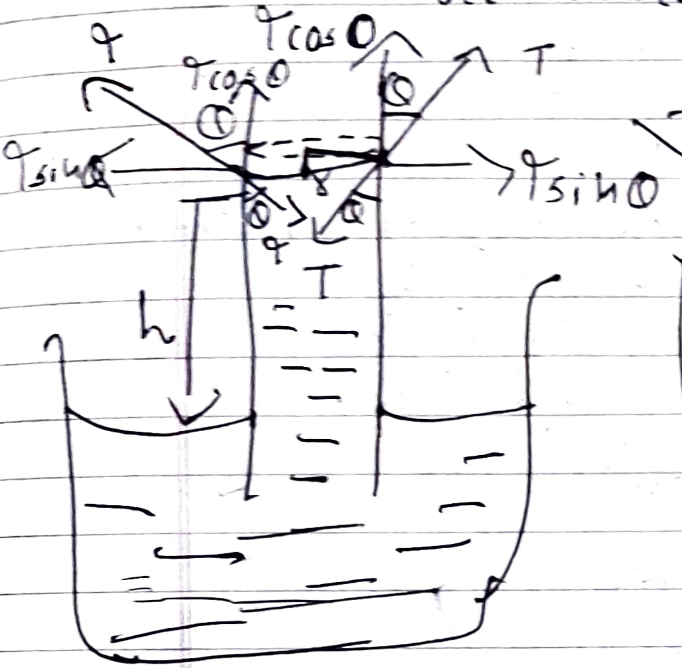
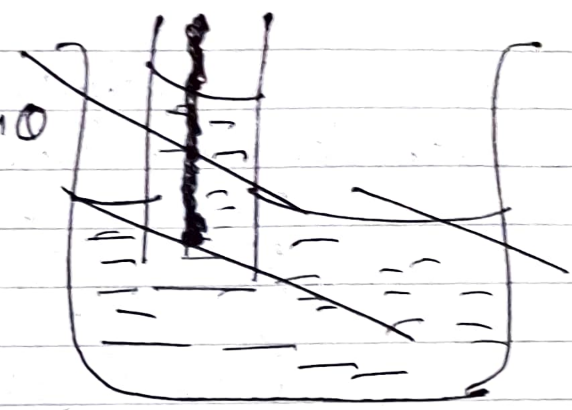


Fig (II): Capillary fall of mercury.

Measurement of surface tension by capillary rise method (Ascent formula)





→ Let us consider open at both end is dipped into a liquid as shown in fig let θ be the angle of contact, T be the surface tension, ρ be density of liquid h be the height to which liquid rises & g be the acceleration due to gravity

Total upward force = $T\cos\theta \times 2\pi r$ — (i)

Volume of liquid rises above the free surface of liquid = Volume of cylinder of height h & radius r + vol of cylinder of height r & radius r − Volume of hemisphere of radius r

$$V = \pi r^2 h + \pi r^2 r - \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$\pi r^2 h + \pi r^3 - \frac{2\pi r^3}{3}$$

$$\pi r^2 h + \frac{\pi r^3}{3}$$

$$\pi r^2 \left(h + \frac{r}{3}\right)$$

$$\pi r^2 h \qquad \text{since } \frac{r}{3} <<< h$$

At equilibrium Condition

Total downward force = Total upward force

$$mg = T\cos\theta \times 2\pi r$$

$$V\rho g = T\cos\theta \times 2\pi r$$

$$\pi r^2 h \rho g = T\cos\theta \times 2\pi r$$

$$h = \frac{2T\cos\theta}{r\rho g} \quad \text{--- (i)}$$

which is a scent formula

$$\left[h \propto \frac{1}{r}\right]$$

$$\sigma = \frac{W}{\Delta A}$$

$$W = T \times \Delta A$$

v when ... = 0.08cm

Hydrostatics

Pascal's law of pressure:- When pressure is applied in enclosed liquid, the pressure is transmitted equally to every portion of it.

Upthrust (Buoyancy):- The upward force experienced by an obj when it is completely or partially emerged in a fluid is called upthrust or buoyancy.

When an obj is completely emersed in a fluid the pressure at its bottom is more than, pressure at its top. due to this diff in pressure upthrust is produced in the liquid

let W_1 be the weight of obj in air & W_2 be the wt of obj in water then upthrust

$$= W_1 - W_2$$

Upthrust = loss in wt of obj in water.

Archimedes' principle: When a body is fully or partially immerged in a liquid it exp an upthrust which is equal to wt of the fluid displaced.

i.e:-

upthrust = Wt of fluid displacement

also

loss in wt of obj in fluid = wt of fluid displaced by obj

law of floatation:

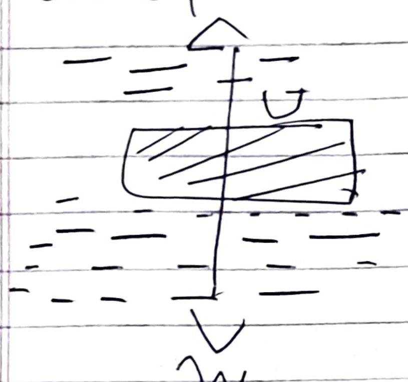


Fig (1)

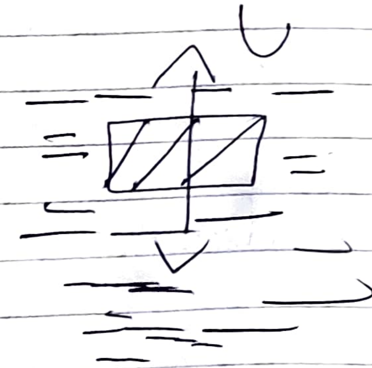


Fig (II)

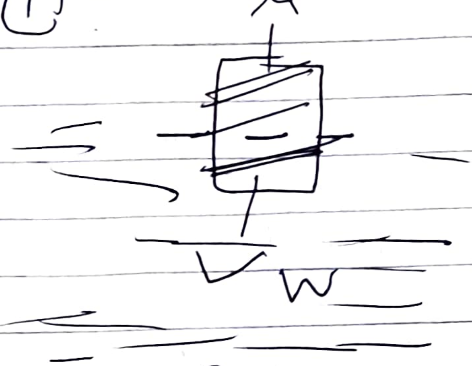


Fig III

Let W be the weight of obj & U be the upthrust due to displaced fluid. If

(i) $W > U$

The body will sink in a liquid as shown in fig (1)

(ii) $W = U$

The body is sinks & in liquid as shown in remain inside liquid with its upper near the liquid surface.

(iii) If $W < U$ The body will float as in fig II

CB Center of Buoyancy

Equilibrium of floating body

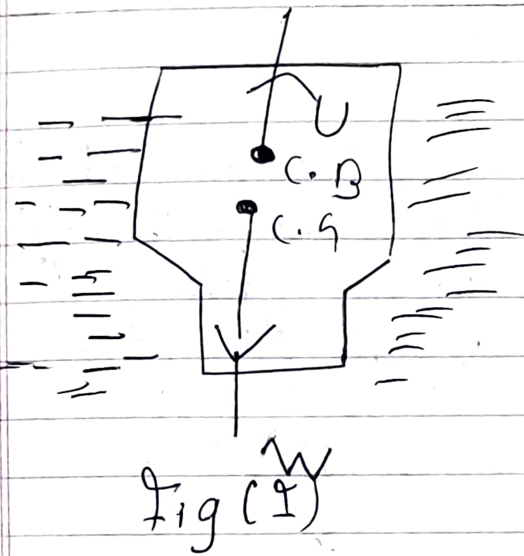


Fig (I)

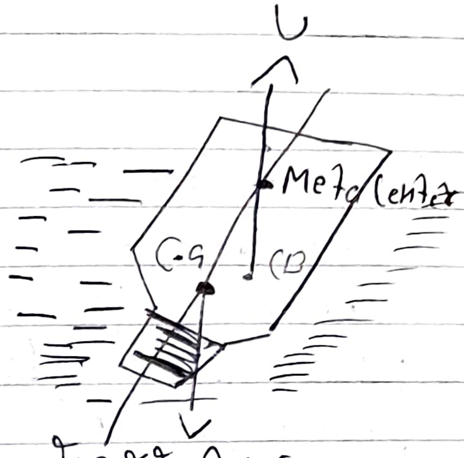
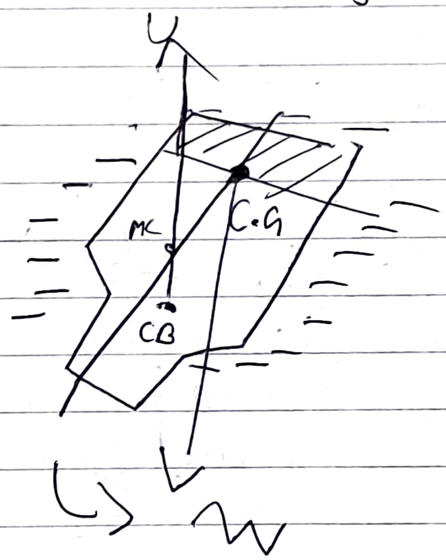


Fig II



Centre of buoyancy is defined as the point of which centre of gravity of the displaced liquid lie.

Metacenter of the floating body is defined as the point of intersection of vertical line passing through the centre of buyancy & original vertical line.

If the floating body is in equilibrium position, the CG of body & CB of displaced fluid lies on the same vertical axis as shown in fig I tilt

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If the floating body is slightly ~~tilted~~ tilted from its equilibrium position, there will be two possible cases.

(1) If the CG of body lies below the C.B then MC will lie above the CG as shown in fig II. The body is then acted by forces W & U at point CG. & CB resp. These two forces form a couple & this couple tends to bring back the body to its equilibrium position.

So the body gains its stability.

(2) If the CG of body lie above the CB then MC will lie below the CG as in fig - III. The body is then acted by forces W & U which forms a couple & this couple rotates more ~~that~~ the body & takes it away from equilibrium position. So, the stability of the body is lost.

273
3

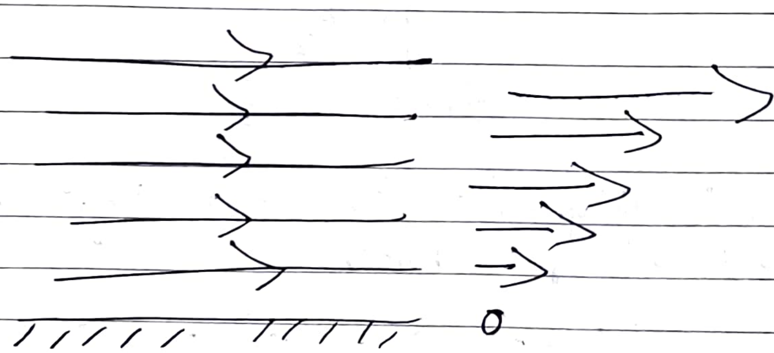
The η of carnot engine working between steam point & ice point
(100 - 0)

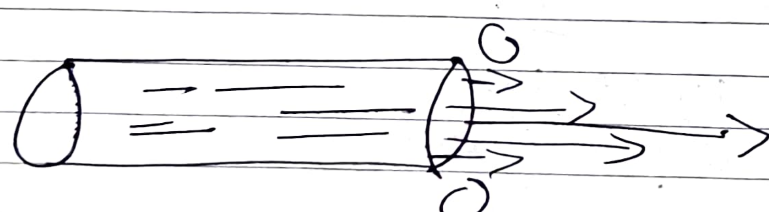
$$1 - \frac{373}{273}$$

VISCOSITY

The property of liquid by virtue of which an opposing force develops between the layers of liquid when there is relative motion between the layers of liquid is called viscocity.

The opposing force that appears between the layers of liquid when there is relative motion between the layers of liquid is called viscous force.

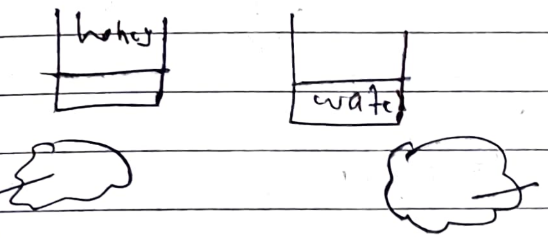




Causes of viscosity

(a) Liquid :- In liquid, viscosity is caused by the cohesive force between the molecules.

(b) Gas :- In gas, viscosity is caused by the collision between the gas molecules.



Spread slowly
More opposing force
More viscocity

Spread fast
less opposing force
less viscosity

Factors affecting viscosity

1. Nature of liquid :-
2. Temp^r :- VVI

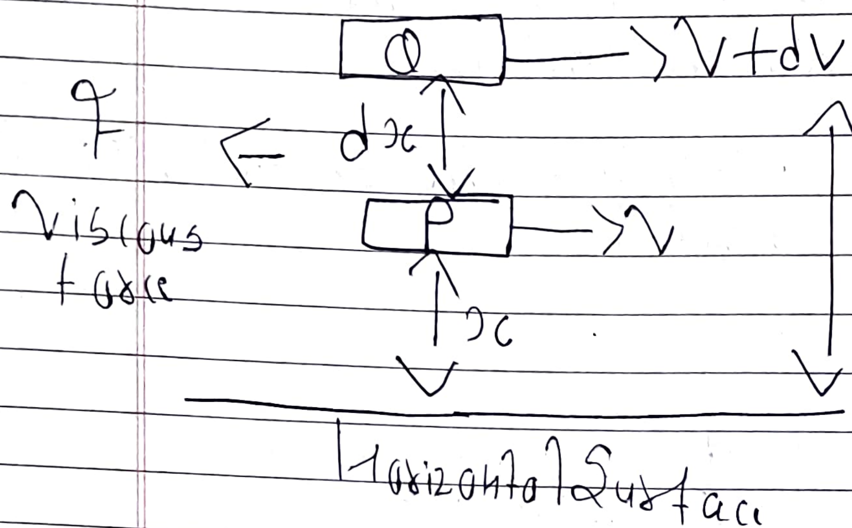
(a) Liquid
As temp^r increases the intermolecular distance between the two layers increases decreasing the cohesive force. Hence viscosity decreases.

(b) Gas
As temp^r increases, the velocity of gas molecules increases increasing the no of collision. Hence viscosity increases.

3. Pressure
On increasing the press^r the viscosity of liquid & gas increasing.

VVI

Newton's formula for viscosity



Let us consider that liquid is flowing on a horizontal surface. Suppose two layers P & Q each of area A are moving with velocities v & $v+dv$ resp. x & $x+dx$ are the respective distances of the layers from the horizontal surface. Let $\frac{dv}{dx}$ be the velocity gradient between the layers of the liquid.

According to Newton, viscous force (F) appearing between layers of liquid is

(i) Directly proportional to the area of each layers.
i.e, $F \propto A$ — (i)

(ii) Directly proportional to the velocity gradient
i.e $F \propto \frac{dv}{dx}$ (ii)

Combing (i) & (ii)

$$F \propto A \frac{dv}{dx}$$

$\frac{m/s^2}{m}$

~~Continuing~~

$$F = -\eta A \frac{dv}{dx} \quad \text{--- (iii)}$$

This is Newton's formula for viscosity.
Here η is a prop constant called coeff of viscosity. Its value depends on nature of liquid & tempr.
$-$ve sign indicates that the direction of viscous force is opposite to the direction of motion of liquid.

From eqn (ii)

$$\eta = \frac{-F}{A \cdot \frac{dv}{dx}}$$

if $A = 1m^2$ & $\frac{dv}{dx} = 1s^{-1}$

Thus, coeff of viscosity (η) can be defined as viscous force appearing between two layers each of unit area & the velocity gradient between them is unity.

Unit of η

$$F = -\eta A \frac{dV}{dx}$$

$$\eta = -\frac{F}{A \cdot dV/dx}$$

ⓐ Cgs unit

$$\eta = \frac{dyne}{cm^2 s^{-1}}$$

$$= 1\, dyne\, cm^{-2} s = 1\, poise$$

ⓑ SI unit

$$\eta = \frac{N}{m^2 s^{-1}} = 1 N m^{-2} s = 1\, decapoise$$

Relation between decapoise & poise

1 decapoise

$= 1 N m^{-2} s$

$(1N)(1m)^{-2}(1s)$

$(10^5 dyne) (100 cm)^{-2}\ 1s$

$10^5 \cdot 10^{-4}\ dyne\ cm^{-2} s$

$10\ dyne\ cm^{-2} s$

10 poise

$\therefore$ 1 decapoise $= 10$ poise

* Dimensional formula of η

$$\eta = \frac{-F}{A \cdot \frac{dv}{dx}}$$

$$\frac{[MLT^{-2}]}{L^2 [T^{-1}]}$$

$[ML^{-1}T^{-1}]$

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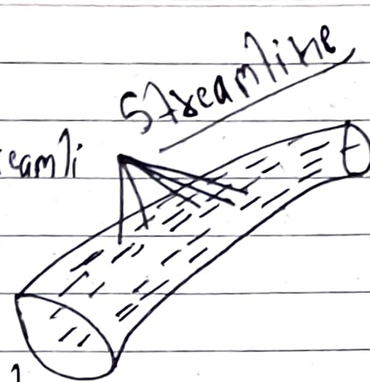
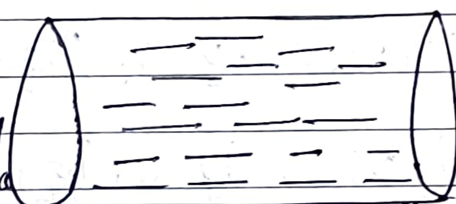
* Streamline motion



The flow of liquid is in streamline motion if each particle of liquid has same magnitude & direction of velocity while crossing a point as that of the preceeding particle while passing through the same point.

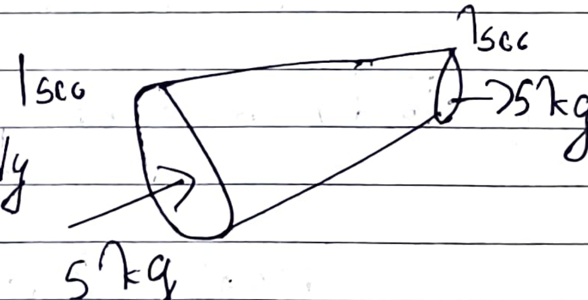
Streamline are path travelled by liquid in streamline flow. Tangent drawn at a point gives the direction of velocity of liquid at that point.

Laminar flow :-



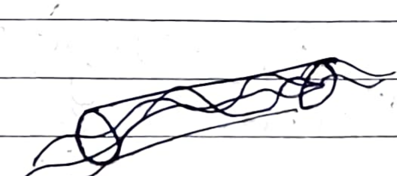
→ If the liquid is flowing over a horizontal surface with a steady flow & moves in the form of layers of different velocities which don't mix with each other then the flow of liquid is called laminar flow. In general laminar flow is a streamline flow.

Steady flow :-



→ The flow of liquid is steady if mass per sec of liquid entering the tube is equal to mass per sec of liquid leaving the tube

Turbulent motion



→ If the velocity of the liquid becomes greater than critical velocity flow becomes irregular & unsteady. This type of motion of liquid is called turbulent motion.

Critical velocity (V_c): The velocity of liquid upto which liquid is in streamline motion & above which liquid is in turbulent motion is called critical velocity.

	8 m/s	streamline
	9 m/s	"
Critical vel –	10 m/s	"
	11 m/s	turbulent

The critical velocity of liquid (V_c) depends on

(a) Coeff of viscosity of liquid
i.e, $V_c \propto \eta^a$ — (i)

(b) Density of liquid
i.e $V_c \propto \rho^b$ — (ii)

(c) Radius of tube
i.e $V_c \propto r^c$ — (iii)

Combining (i), (ii) & (iii)

$$V_c \propto \eta^a \rho^b r^c$$

$$V_c = k \eta^a \rho^b r^c \quad \text{— (iv)}$$

Where
k is a dimensionless constant

$M^0L^1t^{-1}$ $\frac{x}{t}$ $\rho = \frac{M}{V}$ $\frac{M^1}{L^3}$

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$$[M^0L^1T^{-1}] = [M^{-1}L^{-1}T^{+1}]^a [M^1L^{-3}T^0]^b [M^0L^1T^0]^c$$

$$M^0L^1T^{-1} = [M]^{a+b}[L]^{-a-3b+c}[T]^{-a}$$

$a+b=0$, $-a=-1$

$a=-b$ $\quad$ $a=1$

$a=1, b=-1, c=-1$

Putting these values in (V)

$$V_c = k\eta\rho^{-1}r^{-1}$$

$$V_c = \frac{k\eta}{\rho r}$$

Significance of Reynold no.: The above formula for critical velocity is called Reynold's formula & the constant k is called Reynold's no. for liquid to be in streamline motion η should be large, ρ & r should be small. For narrow tube $k = 1000$.

After exceeding this value flow becomes turbulent.

Poiseuille's Formula

Statement

"For streamline & steady flow of a liquid, the volume per second of liquid passing through a uniform & horizontal tube is given by

$$V = \frac{\pi P r^4}{8\eta l}"$$

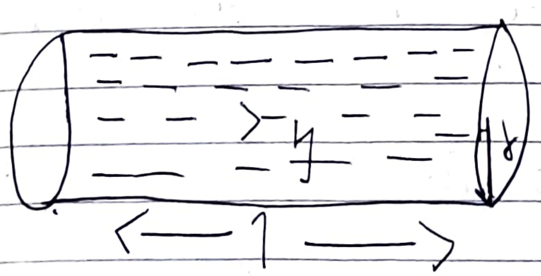
where V is volume per second of liquid passing through the tube

r = radius of tube

l = length of tube

η = coeff of viscosity of liquid.

Proof:



$P = P_1 - P_2$

Let us consider a liquid having coeff of viscosity η is passing through a uniform & horizontal tube of length 'l' & radius r in streamline & steady flow. Let P be the pressure diff between the two ends of the tube.

Poiseuille observed that the volume persecond of liquid (V) passing through the tube depends on

(a) Pressure gradient.

i.e. $V \propto \left(\frac{P}{l}\right)^a$ — (i)

(b) Coeff of viscosity of liquid

$V \propto \eta^b$ — (ii)

(c) Radius of the tube

$V \propto r^c$ — (iii)

Combing

$$V \propto \left(\frac{P}{l}\right)^a \eta^b r^c$$

$$V = k\left(\frac{P}{l}\right)^a \eta^b r^c \quad \text{— (iv)}$$

Where k is dimensionless const

$P = \frac{F}{A}$

$$[L^3T^{-1}] = [ML^{-2}T^{-2}]^a [M^0 L T^0]^c [ML^{-1}T^{-1}]^b$$

$$L^3T^{-1} = M^{a+b} L^{-2a+c-b} T^{-2a-b}$$

$a+b=0$

$a=-b$

$a=\frac{1}{3}$

$-2a+b=-1$

$-2\times -b+b=-1$

$2b+b=-1$

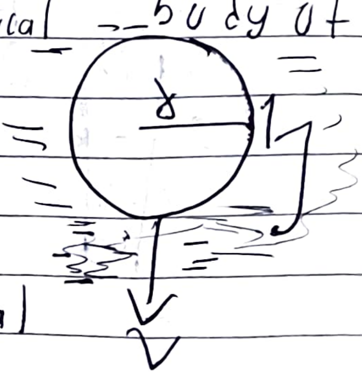
$b=\frac{-1}{3}$

$-2a+c-1$

$a+b=0$
$-2a-b+c=3$
$-2a-b=-1$
solving,
$a=1, b=-1, c=4$
So (iv) becomes
$$V = k\left(\frac{P}{l}\right)^{1} \eta^{-1} r^{4}$$
$$V = \frac{kPr^4}{\eta l}$$
Experimentally value of k was found to be $\frac{\pi}{8}$. so the above eqn becomes.
$$V = \frac{\pi P r^4}{8 \eta l}$$
which is Poiseuille's formula.

Stokes law:- Stmt:- "The viscous force appearing on a spherical body of radius r moving with velocity v in a viscous medium of coeff of viscosity η is given by $F = 6\pi\eta r v$."

Proof:- let us consider a spherical body of radius r is moving with a velocity v in a viscous medium of coeff of viscosity η.



Stokes observed that viscous force (F) appearing on spherical body depends on

(a) Coeff of viscosity of liquid
i.e $F \propto \eta^{a}$ — (i)

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(b) Radius of spherical body

$F \propto r^b$ — (ii)

(c) Velocity of a body

$F \propto v^c$ — (iii)

Combing i, ii, iii

$F \propto \eta^a r^b v^c$

$F = k \eta^a r^b v^c$ — (v)

Where is dimensional less constant.

Writing dimension on both the sides of eqn (v)

$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$

$[MLT^{-2}] = [M^a \quad L^{-a+b+c} \quad T^{-a-c}]$

Comparing the power on both the sides.

$a = 1$,

$-a-c = -2$

$-1-c = -2$

$-1+2 = c$

$c = 1$

Ily $b = 1$

So eqn (v) becomes

$F = k\eta^1 r^1 v^1$

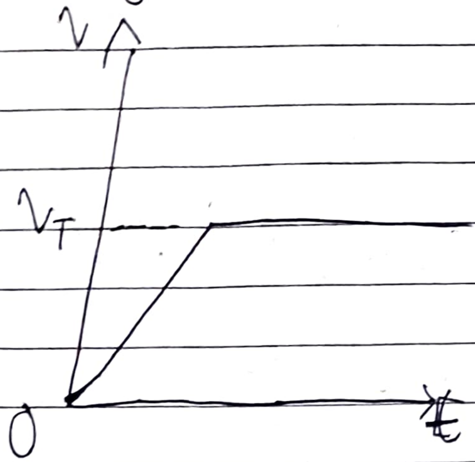
$F = k\eta r v$

Experimentally the value of k was found to be 6π. So the above eqn becomes.

$F = 6\pi \eta r v$

Which is stoke's law.

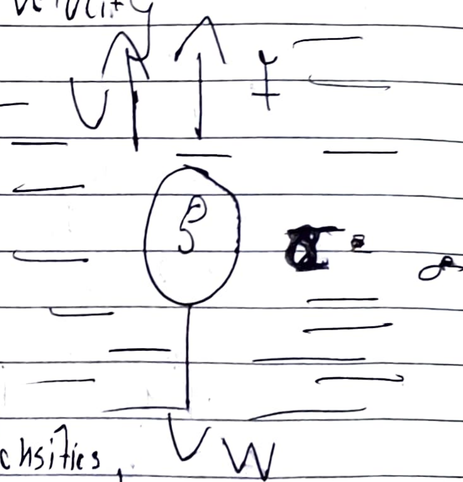
Terminal Velocity (V_T)



→ The maximum uniform velocity acquired by a body when falling freely in a viscous medium is called terminal velocity.

Expression for terminal velocity

→ Let us consider a spherical body of radius r is moving with velocity V_T in a viscous medium of coeff η.



let ρ & σ be the densities of spherical body & viscous medium resp.
When a body is falling freely in a viscous medium, it experiences 3 types of forces.

1. Weight of the body (w) ↓
2. Upthrust (U) ↑
3. Viscous force (F) ↑

(1) During free fall
Upward Force = Downward force

$$f + U = W$$

$$f = W - U$$

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

From the above expression, terminal velocity of the body can be determined.

Numerical (1)
Calculate the terminal velocity of a steel ball of radius 1mm falling freely in oil of coeff of viscosity 2.42 Nsm^{-2} & density 940 kgm^{-3}, taking the density of steel as 7800 $kg\,m^{-3}$ ($g = 10\,ms^{-2}$)

Soln,
radius of body (r) = 1mm = 10^{-3} m
Coeff of viscosity of oil (η) = 2.42 Nsm^{-2}
Density of oil (σ) = 940 kgm^{-3}
Density of body (ρ) = 7800 $kg\,m^{-3}$
$g = 10ms^{-2}$
Terminal velocity $(v_T) = ?$

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$= \frac{2\times(10^{-3})^2(7800-940)\times 10}{9\times 2.42}$$

$$= 6.3\times 10^{-3}\ m/s$$

~~Terminal Velocity (V_T) = ?~~

Numerical-2: Calculate the magnitude & direction of air bubble of radius 1mm moving in oil of coeff of viscosity $0.2\ Nsm^{-2}$ & density $900\ kgm^{-3}$ taking the density of air as $1.29\ kgm^{-3}$. ($g = 10 m/s^2$)

→ Sol^n

Radius of air bubble (r) = 1mm = 10^{-3} m
density of oil (σ) = $900\ kgm^{-3}$
Density of air bubble (ρ) = $1.29\ kgm^{-3}$
Coeff of viscosity (η) = $0.2\ Nsm^{-2}$
$g = 10 ms^{-2}$
$V_T = ?$

$$V_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$= \frac{2 \times (10^{-3})^2 (1.29 - 900) \times 10}{9 \times 0.2}$$

$$= -0.0099 m/s$$

Dir^n → Vertically upward

Equation of Continuity

→ Statement "For streamline & steady flow of an ideal liquid, the velocity of liquid is inversely proportional to the area of cross section of the tube."

i.e. $$V \propto \frac{1}{A}$$

$$V = \text{constant} \cdot \frac{1}{A}$$

$$AV = \text{constant}$$

$$A_1V_1 = A_2V_2$$

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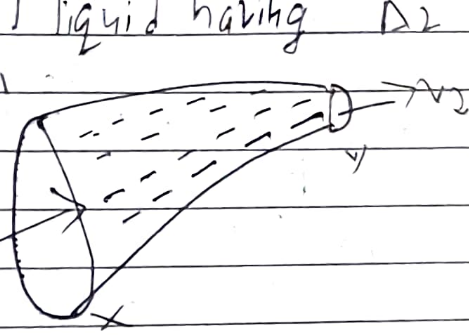
Proof: Let us consider an ideal liquid having density ρ is passing through a non-uniform tube XY in streamline & steady flow.



Let A_1 & A_2 are the area of CSA of the ends X & Y of the tube respectively.
Let v_1 be the velocity of liquid entering the tube & v_2 be the velocity with which liquid is leaving the tub.

Mass per second of liquid entering the tube (m_1) is given by ~~m_1 = (vol~~

$$m_1 = \left(\begin{array}{c}\text{Vol per second of liquid entering the tube} \\ \times \text{density}\end{array}\right)$$

$$m_1 = (A_1 v_1) \cdot \rho \quad - (i) \qquad \left[A \cdot V = A \frac{d}{t} = \frac{v}{t}\right]$$

Mass per second of liquid leaving the tube (m_2) is given by

$$m_2 = (\text{Vol per second of liquid leaving the tube}) \times \text{density}$$

$$m_2 = (A_2 v_2)\,\rho \quad - (ii)$$

During steady state

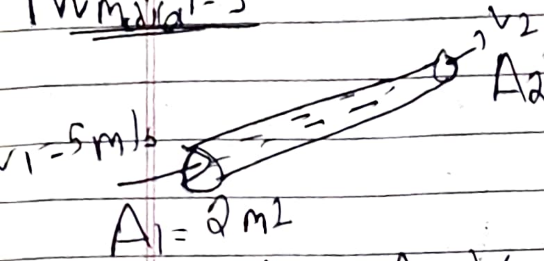
$$m_1 = m_2$$

$$A_1 v_1 \rho = A_2 v_2 \rho$$

$$A_1 v_1 = A_2 v_2$$

which is eqn of continuity.

Numerical-3



$$A_1 V_1 = A_2 V_2$$

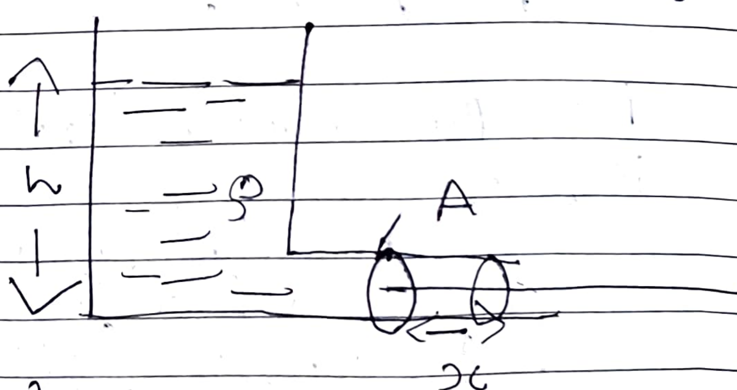
$$\frac{2 \times 5}{0.5} = v_2$$

$$\frac{10}{\frac{1}{2}} = v_2$$

$$v_2 = 20 m/s$$

Energy Possessed by liquid in motion

(a) Pressure energy: The energy possessed by liquid by virtue of its pressure is called pressure energy



Force on the piston

$$F = P.A \quad -(1)$$

Suppose the piston displaces through x

$$W = F.x$$

$$= P.A.x \quad \text{(From eqn-(1))}$$

$$PV \quad (A.x = V)$$

This is the work done by pressure energy.

Pressure energy $= PV$

Pressure energy per unit mass (E_p) is given by

$$E_p = \frac{\text{Pressure energy}}{\text{Mass}} = \frac{PV}{m} = \frac{P}{m/V} = \frac{P}{\rho}$$

(b) Potential energy: The energy possessed by liquid by virtue of its position is called P.E.

$$P.E = mgh$$

Potential energy per unit mass ($E_{P.E}$) is given by

$$E_{P.E} = \frac{P.E}{m} = \frac{mgh}{m} = gh$$

(c) Kinetic Energy: The energy possessed by liquid by virtue of its motion is called K.E.

$$K.E = \frac{1}{2}mv^2$$

Kinetic Energy per unit mass ($E_{K.E}$) is given by

$$E_{K.E} = \frac{K.E}{m} = \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2$$

The total energy per unit mass (E) is given by

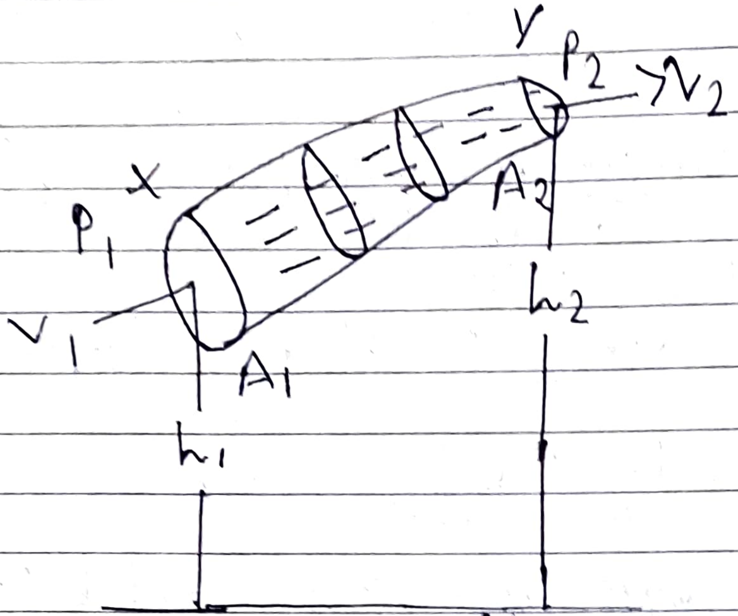
$$E = E_p + E_{P.E} + E_{K.E}$$
$$= \frac{P}{\rho} + gh + \frac{1}{2}v^2$$

Bernoulli's Theorem:

Statement: "For streamline & steady flow of an ideal liquid, the total energy (pressure energy, P.E, K.E) per unit mass remains constant at every cross section of the tube."

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{Constant}$$

Principle of
Based on the ^ conservation of energy.



Horizontal Surface

Let us consider an ideal liquid of density ρ is passing through a non uniform tube in streamline & steady flow. A_1 & A_2 are the area of cross section, h_1 & h_2 are the average height of the ends X & Y of the tube respectively. P_1 & P_2 are the pressure of liquid at ~~xxxx~~ ends X & Y respectively. V_1 is the velocity with which liquid is entering the tube & V_2 is the velocity with which liquid is leaving the tube.

Using eqn of continuity

$$A_1 V_1 = A_2 V_2$$

Since $A_1 > A_2$

So, $V_2 > V_1$

Since liquid is moving up against gravity. So

$$P_1 > P_2$$

Let m be the mass per second of liquid passing through the tube.

The amount of work done per second by liquid entering the tube (W_1) is given by

$$W_1 = P_1 A_1 V_1 \qquad \left[PAV = \frac{F \times d}{t} = \frac{W}{t} \right]$$

The amount of work done per second by liquid leaving the tube (W_2) is given by

$$W_2 = P_2 A_2 V_2$$

The net work done by the liquid is given by

$$W = W_1 - W_2$$

$$P_1 A_1 V_1 - P_2 A_2 V_2$$

$$P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho} \quad \text{--- (i)}$$

$$\left[v = u + at \qquad AV = A\frac{d}{t} = \frac{V}{t} = \frac{m}{\rho} \right]$$

This work done is used to increase the P.E & K.E of the liquid

i.e. $W = mgh_2 - mgh_1 + \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ --- (ii)

Equating eqn (i) & (ii)

$$P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho} = mgh_2 - mgh_1 + \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\frac{P_1}{\rho} - \frac{P_2}{\rho} = gh_2 - gh_1 + \frac{1}{2}v_2^2 - \frac{1}{2}v_1^2$$

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

$$\left[\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant} \right]$$

which is Bernoulli's theorem.

(ρ is constant) $\rightarrow$ $P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

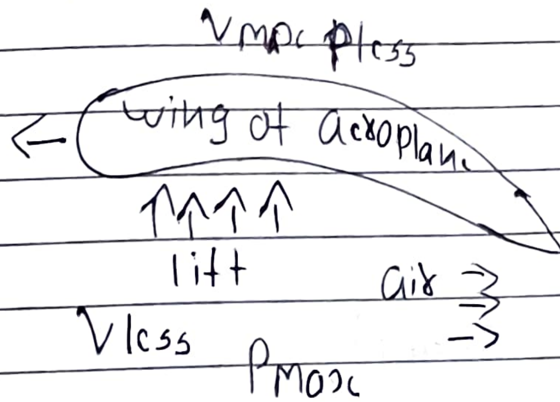
If a tube is horizontal, $h_1 = h_2$

$$\frac{P}{\rho} + \frac{1}{2}\rho v^2 = \text{constant}$$

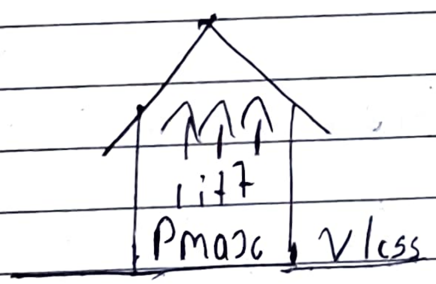
Note :- If v is max P is less.
If v is less P is max.

Application of Bernoulli's theorem:

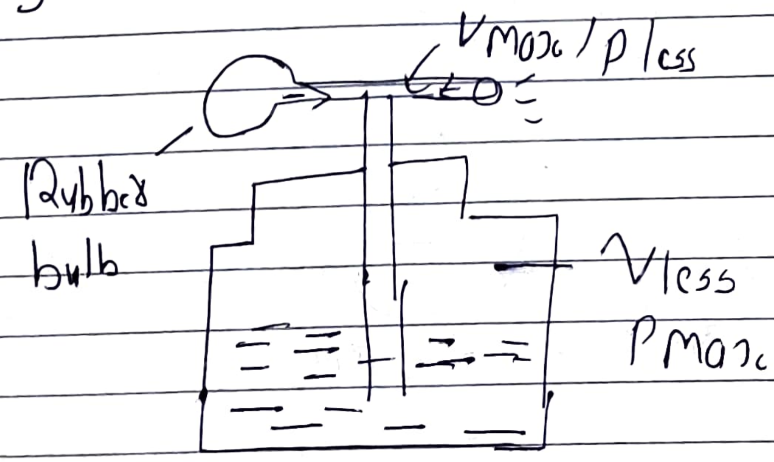
(a) lifting of aeroplane.



(b) flying of roof of a house during strong wind



(c) Spray or atomizer



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(Q) Airport at higher altitude have longer runway, why

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

$\uparrow$ less $\quad$ $\uparrow$ max