Quantisation of Energy

→ Bohr's atomic model:-
Postulates for hydrogen atom:-

(i) Electrons revolve around the nucleus only in certain permitted orbit (circular) of definite radius.

Imp
(ii) Electron can revolve around the nucleus in certain permitted orbit where the angular momentum is integral moment of $\hbar$. i.e

$$mvr = n\hbar$$

$$mvr = \frac{nh}{2\pi} \quad \left[\hbar = \frac{h}{2\pi}\right] = \text{reduced planck's constant.}$$

where $n = 1, 2, 3$ (Principle quantum no.)

Note:- Electron don't radiate energy in this permitted orbit (stationary orbit). (K, L, M, N)

(iii) Electron emits energy when it jumps from higher energy level to lower energy level & vice versa (L-H-absorb).

$$\Delta E = E_2 - E_1$$

E_2 = Energy of higher state

E_1 = lower state Energy

$$\boxed{hf = E_2 - E_1}$$

Radius of n^{th} orbit:-

Bohr assumed hydrogen atom $(Z=1)$ where -vely charged particle $(-e^-)$ revolves around the positively charged Nucleus. $(+e)$

Now, the electrostatic force between these charge at distance 'r' is given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$ [Force of attraction -ve]

[where ε_0 = permettability of free space]

again, The centripetal force required to move the electron in a circular orbit with velocity 'v' is given by

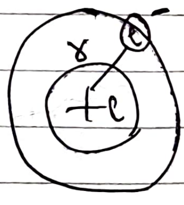
$$F_C = \frac{mv^2}{r}$$

Here, the centripetal force is provided by the electrostatic force ~~of~~ attraction between +e & -e.

Then,

$$F_C = F_e$$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\cancel{r}} = \frac{mv^2}{\cancel{r}\,r^2} \quad \text{--- (i)}$$



$$\cancel{r = \frac{mv^2\, 4\pi\varepsilon_0}{e^2}}$$

also from 2nd postulate,

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$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr} \quad \text{— (ii)}$$

From (i) & (ii)

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{m\, n^2 h^2}{4\pi^2 m^2 r^3}$$

$$\frac{m}{r}\frac{n^2h^2}{\cancel{4}\cancel{\pi}^2 m^{\cancel{2}} \cancel{r^2}} = \frac{1}{\cancel{4}\cancel{\pi}\varepsilon_0}\frac{e^2}{\cancel{r^2}}$$

$$\boxed{r = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}}$$

$$\boxed{r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}}$$

$$\boxed{r_n \propto n^2}$$

For bohr's radius (a_0)

$n = 1$, $r = a_0$

$$a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

$$a_0 = 0.529\,\mathring{A}$$

$$\boxed{r_n = 0.529\,\mathring{A} \times n^2}$$

Velocity in nth orbit

$\frac{mv^2}{r}$

We have,

$$mvr = \frac{nh}{2\pi}$$

$$v_n = \frac{nh}{2\pi m r_n} \quad \text{[Second postulate]}$$

$$V_n = \frac{\cancel{n}\cancel{h}}{2\cancel{\pi}\cancel{m}} \times \frac{\cancel{\pi} m e^2}{\varepsilon_0 n^{\cancel{2}} h^{\cancel{2}}} \quad \text{[Putting value of } r\text{]}$$

$$\boxed{V_n = \frac{e^2}{2\varepsilon_0 h n}}$$

$$v \propto \frac{1}{n}$$

Energy (nth orbit)

Total Energy = K.E + P.E

$$K.E = \frac{1}{2} m v_n^2$$

$$\frac{1}{2} m \times \frac{e^4}{4\varepsilon_0^2 h^2 n^2}$$

$$\frac{1 e^4 m}{8\varepsilon_0^2 h^2 n^2}$$

$$P.E = \frac{1}{4\pi\varepsilon_0} \frac{e}{r} \times (-e)$$

$$\left[-\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r}\right]$$

$$-\frac{1}{4\pi\varepsilon_0} e^2 \times \frac{4\pi^2}{\varepsilon_0 \quad h^2} \quad \frac{me^2\pi}{\varepsilon_0 n^2 h^2}$$

Total Energy $= \frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^2 n^2} - \frac{e^2}{4\pi\varepsilon_0 r}$

$$= -\frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^2 n^2}$$

~~Hydrogen Spectrum~~

[redacted]

~~Principle of physics~~ - 12 Graph

Bohr's Interpretation of Hydrogen Spectrum

→ As the electron jumps from higher energy level to lower energy level it radiates energy in form of electro magnetic wave. The frequency of emmitted radiation is given by

$$hf = E_2 - E_1$$

$$E_2 = -\frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^2 n_2^2}$$

$$E_1 = -\frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^2 n_1^2}$$

$$hf = -\frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^2 n_2^2} + \frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^2 n_1^2}$$

$$hf \; \frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^2}\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}$$

$$f = \frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^3}\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}$$

~~Also, $f = \frac{1}{\lambda}$~~

$v = \lambda f$

$\lambda = \frac{v}{f}$

Also wave no → $\bar{f} = \frac{1}{\lambda}$

$$\frac{f}{c} = \frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^3 c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here,

$$\frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^3 c} = R \text{ [Rydberg Constant]}$$

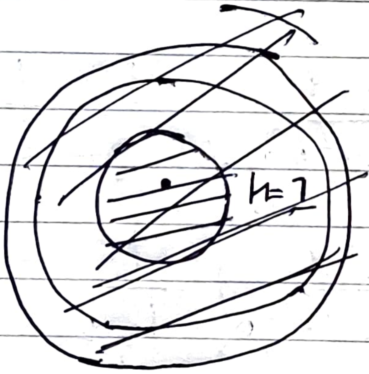
$$R = 1.097 \times 10^7 \ m^{-1}$$

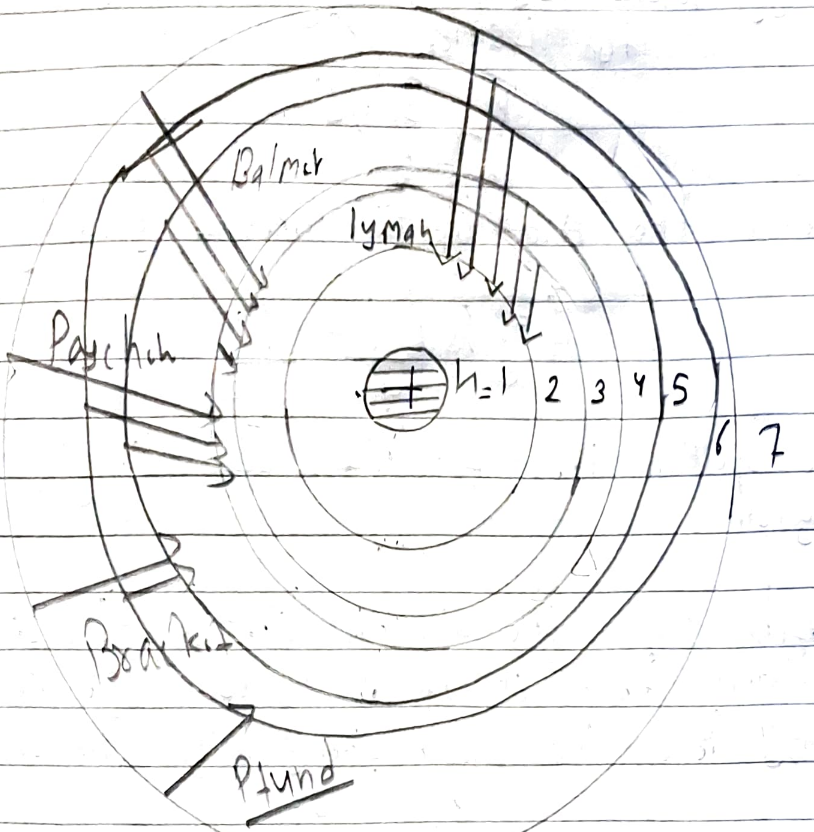
$$\bar{f} = \frac{1}{\lambda} = \frac{f}{c}$$

$$R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Hydrogen Spectrum:

When the electron jumps from higher energy level to the lower energy level it radiates energy which is called spectra.





(1) lyman series: The spectral lines of this series is formed when the transition of the electron takes place from higher energy level to ground state level.

For lyman series,

$n_2 = 2, 3, 4, \ldots$

$n_1 = 1$

Hence, the ~~freq~~ wave no of the lines is given by,

$$f = R = \left(1 - \frac{1}{n_2^2}\right)$$

The spectral lines of this series lies in the UV region.

(2) Balmer Series: The spectral lines of this Series is formed when the transition of electron takes place from higher energy level to ~~lower~~ energy level (i.e $n=2$)

For Balmer,

$$n_2 = 3, 4, 5$$

$$n_1 = 2$$

Hence, the freq of the lines is given by

$$\bar{f} = R\left(\frac{1}{4} - \frac{1}{n_2^2}\right)$$

The spectral lines of this Series lies in the visible region

(3) Paschen series: The spectral lines of this series is formed when transition of e^- takes place from higher energy level to lower energy level (i.e $n=3$)

$$n_2 = 4, 5 \ldots$$

$$n_1 = 3$$

Hence, freq of lines is given by

$$\bar{f} = R\left(\frac{1}{9} - \frac{1}{n_2^2}\right)$$

Spectral lines of this Series lies in IR region

(4) Bracket series: The spectral lines of this series is formed when transition of e^- takes place from higher level to low energy level ($n=4$)

$$n_2 = 5, 6, \ldots$$

$$n_1 = 4$$

Hence, freq of lines is given by,

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$$f = R\left(\frac{1}{16} - \frac{1}{n_2^2}\right)$$

(5) P-fund Series: The spectral lines of this series is formed when transition of e^- takes place from higher energy level to lower energy level (i.e $n = 5$)

For P-fund series

$n_2 = 6, 7, 8,$

$n_1 = 5$

Hence the freq of the lines is given by

$$f = R\left(\frac{1}{25} - \frac{1}{n_2^2}\right)$$

Energy state.

→ As we know total Energy $(E) = -\frac{1}{8}\frac{me^4}{n^2h^2\varepsilon_0^2}$

$= -\frac{13.6}{n^2}$ eV

For $n = 1$

$E_1 = -13.6$ eV

$n = 2$, $E_2 = -3.4$ eV

$n = 3$, $E_3 = -1.51$ eV

$n = 4$, $E_4 = -0.85$ eV

..... $n = 6$, $E_6 = -0.37$ eV

$n = \infty$, $E_\infty = 0$

Excitation energy: It is the minimum req^d energy for the electron to jump from lower energy state to higher energy state.

For eg :- Energy reqd to jump from $n=1$ orbit to $n=2$ orbit in hydrogen atom, 10.2eV energy is needed

For H - atom

$$\Delta E = E_2 - E_1$$
$$= -3.4 + 13.6$$
$$10.2\,eV$$

Excitational Potential :- Minimum potential which supply energy to jump the e^- from lower energy state to higher energy state.

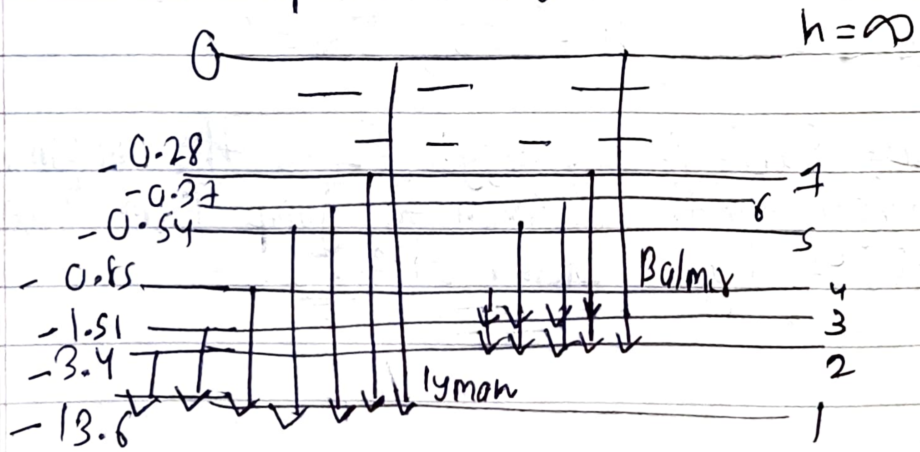
$$E.P = \frac{\text{Excitation Energy}}{e}$$

e = Charge

Ionizing energy :- Minimum energy reqd, to knock out the e^- from the respective atom is called ionizing energy.

Ionization potential :- Minimum potential applied to knock out e^- from atom is called Ionization potential.

Emmission Spectra : H-atom



Bohr's rad, v, E
Bragg's law
De-Broglie's theory - numerical

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De-Broglie Theory: Duality:-

⇒ In 1924 A.D. De-Broglie proposed a theory based on following facts.

(i) In the universe, whole energy is in the form of radiation & matter so, they should have similar characters.

(ii) Nature is symmetric so, if radiation have dual nature then matter also have dual nature.

De-Broglie Wave length.

$E = hf$

→ Acc to the plank's equation, the energy possessed by the radiation is given by

$$E = hf \text{ ——— (i)}$$

where h = plank's constant
f = freq of wave.

Also, if we consider this photon as a particle of mass 'm',

$$E = mc^2 \text{ ——— (ii)}$$

where c = speed of light.

Here, Energy in both case is same, so,

$$hf = mc^2$$

$$h = m\frac{c^2}{f}$$

$v = \lambda f$

$$h = mc\lambda$$

$$\boxed{\lambda = \frac{h}{mc}}$$

$$\boxed{\lambda = \frac{h}{p}}$$

$\lambda = \frac{h}{\cancel{2\pi}}$ $\omega = \tau \times s$ $d\omega = \tau\, dy$

IIly, The wavelength of the matter moving with velocity "v", is given by,

$$\boxed{\lambda = \frac{h}{mv}}$$

Which is the de-Broglie equation.

(a) Calculate the de-broglie λ of e^-:

$$\lambda = \frac{6.62 \times 10^{-27}}{9.1 \times 10^{-27} \times 3 \times 10^{8}}$$

→ Suppose an e^- is accelerated with potential 'V', then gain in K.E is given by

$$K.E = eV$$

$$\frac{1}{2}mv^2 = K.E$$

$$v = \sqrt{\frac{2K.E}{m}}$$

Now,

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{h}{\sqrt{\frac{2KEm^2}{m}}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2E_k m}}}$$

$$\lambda = \frac{h}{\sqrt{2eVm}}$$

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Heisenberg Principle:- It states that, it is impossible to measure the accurate value of position & momentum simultaneously. If you try to measure the accurate value of position then, the value of momentum will be uncertain & vice-versa.

Mathematically

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

Q Explain why e^- cannot lie inside nucleus.

-> Let the electron is in the nucleus, therefore let the position (Δx) of the e^- be the radius of nucleus

we know,

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

$$10^{-15} \Delta p \geq \frac{h}{2\pi}$$

$$\Delta p \geq \frac{h}{2\pi} \times 10^{15}$$

$$\Delta p = 1.05 \times 10^{-19}$$

$$v = \frac{\Delta p}{m}$$

$$v = \frac{1.58 \times 10^{-19}}{9.1 \times 10^{-31}}$$

$$v = 1.157 \times 10^{11} >>> c$$

So e^- cannot lie inside nucleus.

-> If e^- lie inside the nucleus, acc to Heisenberg principle, then speed of e^- must be greater than light which is impossible so, ________ .

(1) Calculate the De-broglie wavelength of an e^- acc through a V of 2kV.

$$\lambda = \frac{h}{\sqrt{2eVm}}$$

$$\frac{6.62\times10^{-34}}{\sqrt{2\times2\times10^{3}\times1.6\times10^{-19}\times9.1\times10^{-31}}}$$

$\cancel{2.74\times10^{-4}\,m}$

$2.74\times10^{-9}\,m$

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$ (For anything other than hydrogen)

$E = E_2 - E_1$

$K.E = -E$

$P.E = \frac{2E}{\cancel{2}}$

Shortest Jump = λ_{max}

Longest Jump = λ_{min}

(1) ~~Calculate the de-Broglie wavelength of~~ The total energy of an e^- is in the first excited state of hydrogen atom is -3.4 eV.

(a) what is the P.E

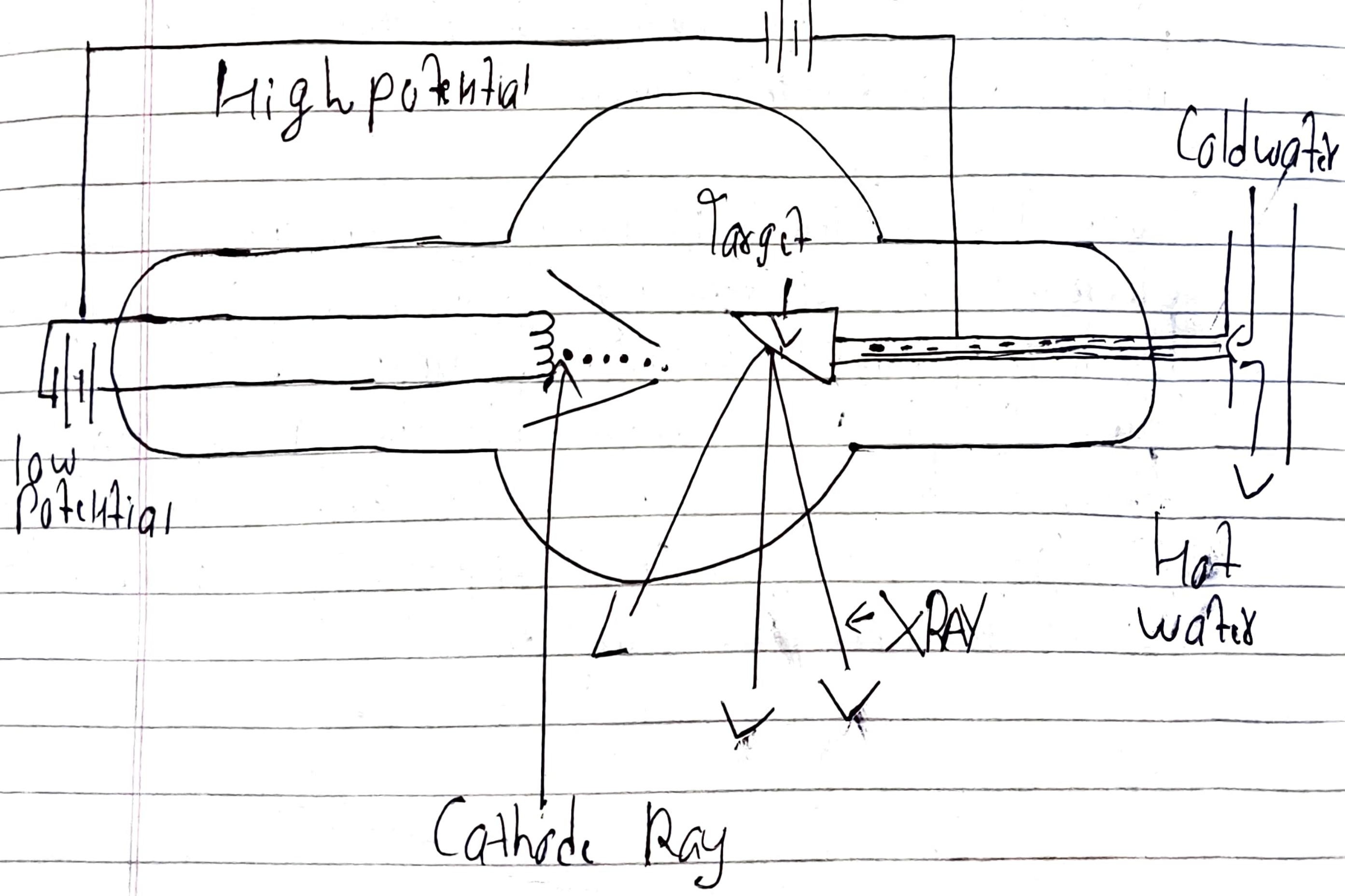
(b) If the e^- in the ground st... in the ... fo the ground st...

Calculate the ...

Quantization Of Energy
↳ XRAY

- It was discovered by german physicist Wilhelm conrad Rontgen in 1895.
- XRAY are electromagnetic wave of very short wavelength lies between 10^{-12} to 10^{-9} m.

II Production Of XRAY:- Coolidge Tube



Quality of xray depends on high pt
Target is of high Atomic weight & Mpt.

XRAY are produced by bombardment of fast moving electron (i.e cathode ray) on the metal target of high atomic weight & melting point.

Colidge tube consists of evaculated glass tube of pressure 10^{-5} mm of Hg having cathode & anode. The cathode consists of tongsten filament having high resistivity which is heated by low tension battery & it is placed inside metal cup to hit the target. The metal target is generally made by using elements of high atomic weight like molybdenum & is ~~(connected)~~ connected to the terminal of higher potential. The target is kept at an angle of 45° to the horizontal. The target is connected to the positive & filament to the -ve of high tension electric source.

When the filament is heated by low tension battery the electrons are emitted by it. These e⁻ are accelerated by high potential between filament & target. When the fast moving electron hit the metal target through the cup x-ray are produced. The water is circulated through the copper tube to the metal target to cool the target.

XRAY

Control of Intensity & Quality of XRA

(i) Control of Intensity:- Intensity of XRAY depends on the no of e^- striking the target metal. The no of e^- depends on current passing through the filament. Hen Intensity of XRAY can be controlled by adjusting filament current.

(ii) Control of Quality:- The freq of XRAY ressemble the quality of XRAY. freq of XRAY depends on voltage beth anode & cathode.

Let 'm' be mass of e^-, 'e' be charge of e^- & V be Pd between electrodes then

Work done by Pd = Max K.E gained by e^-

$$eV = \frac{1}{2} m v_{max}^2 \quad \text{---(1)}$$

Where v_{max} = max vel of e^- striking the target.
If all the K.E of e^- is converted into energy of XRAY then,

$$\frac{1}{2} m v_{max}^2 = hf_{max}$$

$$eV = hf_{max}$$

$$f_{max} = \frac{eV}{h}$$

Then, min wavelength of XRAY

$$\lambda_{min} = \frac{c}{f_{max}}$$

$$\lambda_{min} = \frac{c}{\left(\frac{eV}{h}\right)}$$

$$\lambda_{min} = \frac{hc}{eV}$$

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C = Speed of light
h = Plank's constant
V = acc potential

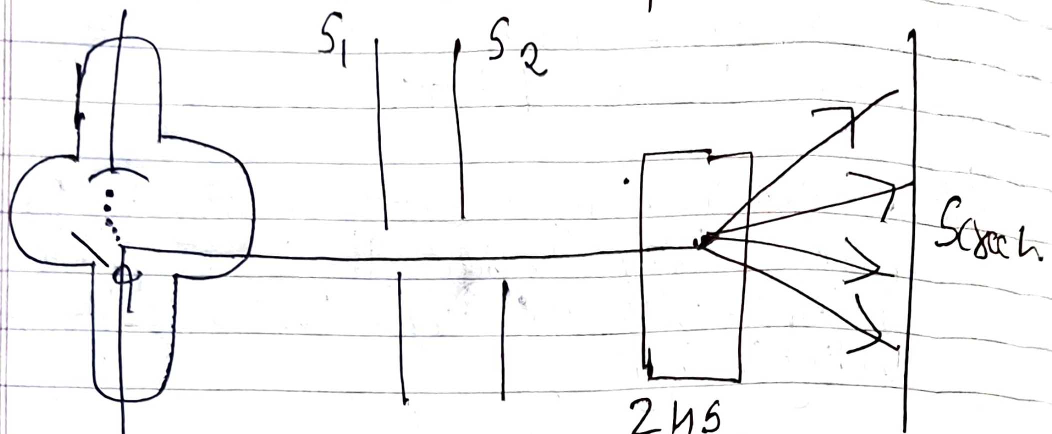
If e⁻ in a x-Ray tube accelerates through the pd of 10kV before striking the target. If an e⁻ produces 1 photon on impact with the target. What is the λ of resulting X-ray?

$V = 10kV$
$h = 6.62 \times 10^{-34} Js$
$e^- = 1.6 \times 10^{-19}$
$C = 3 \times 10^8$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 10}$$

$\lambda = 1.24 \times 10^{-10}$

XRAY diffraction [laues Exp]

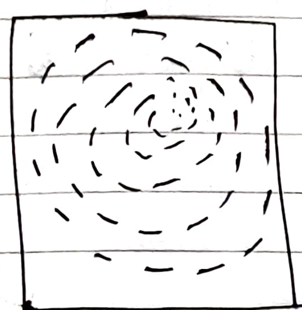


-> The diffraction effect can be observed only if the spacing between the lines ruled on the grating is of order of magnitude of λ of light used.

-> Crystal is used to show the diffraction of Xray. In a crystal grating, the atoms arranged in a regular pattern correspond to the grating lines & the distance betwn two atoms to the grating element. In 1930 laues experimentally showed the diffraction of XRAY through the crystal. A narrow beam of XRAY is passed through two slit S_1 & S_2 & is passed through ZnS crystal & allowded the emerging beam to fall on photographic plate. After exposing for several hours & developing the plate. Many ~~faint~~ faint but regular arranged spots are seen. This spot is called laues's spot & it is due to diffraction of XRAY. This spots are arranged according to different geometrical pattern for different ~~geometrical structure~~ crystal depending upon diff geometrical structure.

-> Following conc were made by the Exp

(1) XRAY are electromagnetic wave.
(2) The atoms of crystal are arranged in regular 3d lattice.
(3)



Developed Photographic plate.

A xray ~~depends~~ tube works at a DC potential diff of 50 kV. Only 0.4 % ~~percent~~ of energy of cathode ray is converted into XRAY & the heat is generated on the target at rate 600 W. Calculate the current passed & velocity of e^- striking the target.

* $[P = V \times I]$

$(m_e = 9.1 \times 10^{-31} kg)$

$P.d = 50 \times 10^3 V$

XRAY = 0.4 % of e^-

$P = 600 W$

$I = \frac{600}{50 \times 10^3}$

$0.012 A$

$\frac{1}{2} mv^2 = eV$

$\frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 = 1.6 \times 10^{-19} \times 50 \times 10^3$

$v^2 = 1.758 \times 10^{16}$

$v = 1.32 \times 10^8$

An xRAY tube operated at DC P.d. of 10 kV produced heat at the target at the rate 720 W, Assuming 0.5% of the incident e^- is converted into xRAY radiation calculate the no of e- striking per second at the target & the velocity of incident electron.

$C \quad \frac{e}{m} = 1.8 \times 10^{11}$ C/kg

$V = 10\,kV$

$10 \times 10^{3}\,V$

$\boxed{P = V \times I}$

$X = 0.5\%$

heat $= 99.5\%$

99.5% of $(10 \times 10^{3} \times I) = 720$

$0.995\,(10^{4} I) = 720$

$10^{4} I = 723.61$

$I = 0.072\,A$

$I = \frac{q}{t}$

$I = \frac{ne}{t}$

$\frac{n}{t} = \frac{I}{e}$

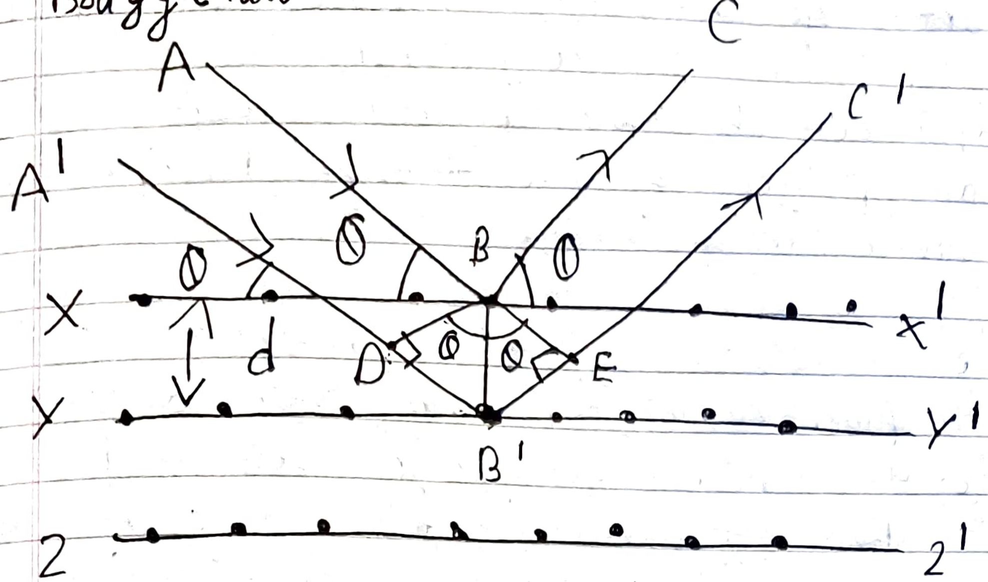
$= \frac{0.072}{1.6 \times 10^{-19}}$

$\frac{n}{t} = 4.522 \times 10^{17}$

$eV = \frac{1}{2} mv^2$

$v^2 = \sqrt{\frac{2eV}{m}} = \sqrt{2 \times 1.8 \times 10^{11} \times 10^{4}}$

Bragg law



From fig:

Total path diff $= B'D + B'E$

In $\triangle BB'D$

$$\sin\theta = \frac{B'D}{BB'}$$

$$\sin\theta = \frac{B'D}{d}$$

$$B'D = d\sin\theta$$

Ily

In $\triangle BB'E$

$$\sin\theta = \frac{B'E}{BB'}$$

$$B'E = d\sin\theta$$

Total path diff $= 2d\sin\theta$ $[d\sin\theta + d\sin\theta]$

For maximum intensity of reflected beam of x-ray,

$$\boxed{2d\sin\theta = n\lambda}$$

$n = 1, 2, 3 \ldots\ldots$ is the order of reflection.

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langpart → The intensity of the reflected beam of X RAY will be maximum when the path diff between two reflected ~~beam~~ wave from diff plane is an ~~intergal~~ integral multiple of wavelength of x-ray.

let us consider a crystal containing a set of parallel plane having the inter plannar distance 'd'. let us suppose the parallel beam of monochromatic xray of wavelength λ is incident on the crystal lattice with the glancing angle θ. let AB be the incident RAY & BC be the reflected ray from xx', Ily A'B' be the incident ray & B'C' be the reflected ray from the plane YY'.